

## Spontaneous fragmentation of topological black holes

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### Abstract

We study the metastability of Anti-de Sitter topological black holes with compact hyperbolic horizons. We focus on the five-dimensional case, an AdS/CFT dual to thermal states in the maximally supersymmetric large- $N$  Yang–Mills theory, quantized on a three-dimensional compact hyperboloid. We estimate the various rates for quantum-statistical D3-brane emission, using WKB methods in the probe-brane approximation, including thermal tunneling and Schwinger pair production. The topological black holes are found to be metastable at high temperature. At low temperatures, D-branes are emitted without exponential suppression in superradiant modes, producing an instability in qualitative agreement with expectations from weakly-coupled gauge dynamics.

# 1 Introduction

Topological black holes may be counted among the most exotic specimens in the quite vast bestiary of black objects studied recently. They are characterized by event horizons of complicated topology, obtained by modding a hyperbolic hyperplane by a freely acting discrete isometry, and exist in an asymptotically Anti-de Sitter spacetime (AdS) with compatible topological identifications [1, 2, 3]. Similar black objects arise naturally in string theory as the near-horizon limit of thermally excited D-branes with such world-volume topology. In particular, for the case of D3-branes one finds topological black holes in  $\text{AdS}_5$  with a direct interpretation in terms of a dual four-dimensional conformal field theory (CFT) living on a compact 3-hyperboloid  $\Sigma_3 = \mathbf{H}^3/\Gamma$ , where  $\Gamma$  is a freely acting discrete isometry.

AdS/CFT for CFTs on spaces of negative curvature has been comparatively less studied, since most physical questions investigated so far can be appropriately formulated in more standard examples of CFTs defined on flat tori or spheres. There are, however, good reasons to study these models in various contexts.

The low-lying spectrum of field theories on spaces of the form  $\Sigma_n = \mathbf{H}^n/\Gamma$  is quite interesting. While the spectral gap of the Laplacian operator on  $\Sigma_n$  is controlled by the curvature radius  $\ell$ , the volume of the compact hyperboloid is determined by the overall size induced by the  $\Gamma$  identification. This means that Kaluza–Klein models based on compact hyperboloids can support hierarchies of couplings with good decoupling properties of KK modes [4]. When such compactification manifolds are contemplated in string theory, new phenomena take place regarding the dynamics of winding modes, beyond the usual rules of T-duality (cf. [5]).

The effect of negative curvature on the stability of CFT’s and their AdS/CFT counterparts has been studied from various points of view (see for example [6, 7, 8, 9]). Any scalar degree of freedom with a conformal coupling has a background curvature coupling of the form (in four dimensions)  $-\phi^2 R/6$ , where  $R$  is the Ricci scalar. This term renormalizes the  $\phi$  mass increasing its stability for positive background curvature, i.e. the case of the sphere for instance. Conversely, for the theory on  $\Sigma_n$  this term induces a tachyonic shift in the effective mass-squared of  $\phi$ . Since this shift is of order  $-1/\ell^2$ , the same order of magnitude as the spectral gap, only a few modes can be tachyonic in practice, sometimes just the zero mode. Our main concern in this note is the study of whether this instability of the zero mode may be converted into *metastability* by thermal effects.

AdS/CFT models for CFT’s on hyperboloids have been motivated recently as an interesting toy model for the holographic study of black hole interiors [10]. By a clever use of different conformal frames, a specific topological black hole on  $\text{AdS}_5$  can be related to a four-dimensional CFT on a Milne-type cosmology with compact hyperbolic spatial sections. Dynamical processes in which black holes are formed by sending D-branes from infinity may be studied in the particular case of the maximally supersymmetric Super Yang–Mills (SYM) theory with  $SU(N)$  gauge group. One sets up the scattering of an initial state with large values of the adjoint Higgs fields into an intermediate resonance of vanishing Higgs fields [11, 10], whose dynamics appears related to the crunch singularity

in the black hole interior. The black hole corresponds to the thermalization of these Higgs fields near the origin of field space, and the subsequent decay of the black hole would complete the S-matrix description of the process.

Our main interest in this note is the decay phase, whereupon the quasi-static black hole ejects the branes back to infinity. We concentrate on the  $n = 3$  case, i.e. topological black holes in  $\text{AdS}_5$  with a dual description in terms of thermal states of the maximally supersymmetric Yang–Mills theory in four dimensions (see [8] for earlier work on this system.) Our results should generalize naturally to all  $n > 1$ , provided a microscopic description of the system is available (such as the simple cases based on M2 or M5 branes). The  $n = 0, 1$  cases pose additional subtleties, even for the theories defined on spatial manifolds of positive curvature, and we shall not discuss them here (see [6, 7]).

## 2 Topological black holes

Let us consider a family of black-hole spacetimes with hyperbolic horizons and  $\text{AdS}_5$  asymptotic metric,

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{\ell^2} d\Sigma_3^2, \quad (1)$$

where

$$f(r) = \frac{r^2}{\ell^2} - 1 - \frac{\mu}{r^2} \quad (2)$$

and  $d\Sigma_3^2$  is the metric of a three-dimensional compact hyperbolic space  $\mathbf{H}^3/\Gamma$  of curvature radius  $\ell$  and volume  $V$ . In what follows we shall use units in which the AdS radius of curvature is set to unity,  $\ell = 1$ . In these units, the minimum value of  $\mu$  compatible with regularity is  $\mu_{\min} = -1/4$ , which defines the extremal black hole, whereas the  $\mu = 0$  case has the special property of being locally isometric to pure AdS. The associated Hawking temperature is  $T = \beta^{-1}$  with

$$\beta = \frac{2\pi r_0}{2r_0^2 - 1}, \quad (3)$$

and

$$r_0^2 = \frac{1}{2} \left( 1 + \sqrt{1 + 4\mu} \right) \quad (4)$$

determines the horizon radius. The energy of these black holes reads

$$M = \frac{3V}{16\pi G} \left( \mu + \frac{1}{4} \right) = \frac{3V}{16\pi G} \left( \frac{1}{4} + r_0^4 - r_0^2 \right), \quad (5)$$

whereas the entropy takes the usual form

$$S = \frac{V}{4G} r_0^3, \quad (6)$$

with  $G$  denoting the effective five-dimensional Newton constant. Using the standard dictionary to parameters of the dual  $SU(N)$  gauge theory we have (cf. [2])

$$\begin{aligned} M(N, T) &= \frac{3\pi^2 N^2}{32} a(T)^2 V T^4 , \\ S(N, T) &= \frac{\pi^2 N^2}{16} a(T)^3 V T^3 , \end{aligned} \tag{7}$$

where we denote

$$a(T) \equiv 1 + \sqrt{1 + \frac{2}{(\pi T)^2}} .$$

The free energy  $F = M - TS$  is then given by

$$F(N, T) = -T S(N, T) \left( 1 - \frac{3}{2a(T)} \right) , \tag{8}$$

and the associated chemical potential for the Ramond–Ramond (RR) charge

$$\mu_N \equiv \frac{\partial F}{\partial N} = -\frac{1}{N} T S(N, T) \left( 2 - \frac{3}{a(T)} \right) \tag{9}$$

is negative at all temperatures and vanishes in the zero-temperature limit. In this form, the thermodynamic functions refer to thermal states of the  $SU(N)$ , maximally supersymmetric Yang–Mills theory quantized on  $\Sigma_3$ , in the limit of large  $N$  and large 't Hooft coupling,  $N \gg \lambda \gg 1$ , where  $\lambda \equiv g_{\text{YM}}^2 N$ .

The thermodynamics implied by these expressions is standard in the large temperature limit, but it is quite peculiar for very low temperatures. As pointed out in [2], the zero-energy configuration, corresponding to the  $T = 0$  black hole, retains a macroscopic entropy of order  $N^2$ . This seems to be a radical effect of the strong coupling limit, implicit in the supergravity description.

The specific heat of these black holes is positive, and so they can come to equilibrium with radiation trapped in AdS. This means that these black holes are stable with respect to radiative decays through particle degrees of freedom in the supergravity multiplet. Again, the positivity of the specific heat persists down to zero temperature, so that there are no perturbative signs of any instabilities for the highly degenerate zero-energy level.

The black holes themselves can be viewed as thermal states in the gauge theory with all the adjoint Higgs fields concentrated around  $\phi = 0$ . Such a configuration should suffer from instabilities because of the classical tachyonic potential  $V(\phi)_{\text{cl}} \propto -\phi^2$  of the Higgs zero mode on  $\Sigma_3$ . Still, the Higgs field can be locally trapped by thermal effects near the origin of field space provided the temperature is large enough, i.e. for  $T \gg 1$ , in units of the curvature radius of  $\Sigma_3$ , the effective potential obtained by integrating out non-zero modes in a thermal state around  $\phi = 0$  should contain a standard thermal mass of the form  $m_\beta^2 \sim \lambda T^2$ , which overcomes the tachyonic term induced from the background curvature for  $\beta \ll \sqrt{\lambda}$ , whereas at large field strengths the supersymmetric nature of the

high-energy theory should yield temperature-dependent effects in the quantum effective potential suppressed by powers of  $T/\phi$ . Under these circumstances we can expect the classical negative quadratic term to dominate at large  $\phi$ , thus depicting an unbounded effective potential with a local minimum at  $\phi = 0$ . For very low temperatures the tachyonic mass dominates even in the region of small fields, and we expect the dynamics to be described by a classical runaway of the Higgs fields down the tachyonic potential.

A full calculation of this effective potential to leading order in perturbation theory in the SYM theory is an interesting open problem. In this note we shall perform a calculation motivated by the strong-coupling description of the system as a topological black hole on AdS, by considering the fragmentation of the black hole by D3-brane emission. We shall work in the brane-probe approximation and consider the Hawking radiation of a single D3-brane, with the subsequent quantum-statistical process of decay into the asymptotic runaway region. In the SYM interpretation, we are then considering the spontaneous decay of the thermal state at the origin of field space, via the symmetry breaking pattern  $SU(N) \rightarrow SU(N-1) \times U(1)$ .

### 3 Brane probes

In the brane picture of the breaking pattern  $SU(N) \rightarrow SU(N-1) \times U(1)$ , the probe-brane effective action is identified with the result of integrating out all the  $SU(N-1)$  degrees of freedom, and the corresponding thermal effects incorporated into the temperature dependence of the effective action. In turn, this temperature dependence enters the probe-brane calculation via the  $\mu$  parameter in the black hole metric (see for instance [12]). In our analysis, we will obviate the dynamics with respect to the R-symmetry group, i.e. the motion of the D3-branes in the ‘internal’  $\mathbf{S}^5$ , so that we only consider the rigid motion of D3-branes in  $\text{AdS}_5$ . We also freeze to their vacuum values the excitations of the world-volume gauge fields in our probe brane.

Under such conditions, we take the world-volume  $\mathcal{W}$  of the probe D3-brane to be the embedding into  $\text{AdS}_5 \times \mathbf{S}^5$  of  $\mathbf{R} \times \Sigma_3 \times P$ , where  $P$  is a fixed point on  $\mathbf{S}^5$ ,  $\mathbf{R}$  represents time and the whole  $\Sigma_3$  is mapped rigidly at the same radial position in the coordinates of (1), given by a function  $r(t) \geq r_0$ . The probe effective action takes the form

$$I = -\mathcal{T}_{\text{D3}} \int_{\mathcal{W}} \sqrt{-\det(h_{ab})} + \varepsilon \mathcal{T}_{\text{D3}} \int_{\mathcal{W}} C_4, \quad (10)$$

where  $h_{ab}$  is the induced metric on the world-volume and  $C_4$  is the RR four-form coupling minimally to the D3-brane. D3-branes correspond to  $\varepsilon = +1$ , whereas  $\varepsilon = -1$  yields the action of an antiD3-brane.

A natural additive normalization for  $C_4$  is obtained by requiring it to be well-defined on the Euclidean black-hole spacetime, with topology  $\mathbf{R}^2 \times \Sigma_3 \times \mathbf{S}^5$ , with the Euclidean time being an angular variable in the  $\mathbf{R}^2$  factor. In this case the projection of  $(1/4)dC_4$  onto the AdS factor is the volume form of the Euclidean AdS black-hole, and the Euclidean action takes the Wess–Zumino form

$$I_E = \mathcal{T}_{\text{D3}} \text{Vol}[\mathcal{W}] - 4\varepsilon \mathcal{T}_{\text{D3}} \text{Vol}[\overline{\mathcal{W}}], \quad (11)$$

with  $\overline{\mathcal{W}}$  the compact four-dimensional manifold having  $\mathcal{W}$  as its boundary. This prescription demands that the AdS projection of the RR form be given by

$$C_4|_{\text{AdS}} = (r^4 - r_0^4) d\Sigma_3 \wedge dt , \quad (12)$$

so that it vanishes at the horizon in the static frame.

Evaluating the action for the particular embedding under consideration we find, in the real-time static coordinate system

$$I = -m \int dt \left[ r^3 \sqrt{f(r) - \frac{\dot{r}^2}{f(r)}} - \varepsilon (r^4 - r_0^4) \right] , \quad (13)$$

where  $\dot{r} \equiv dr/dt$  and we introduce an ‘effective mass parameter’

$$m \equiv V \mathcal{T}_{\text{D3}} = \frac{NV}{2\pi^2} , \quad (14)$$

in units  $\ell = 1$ .

We can obtain a first indication of the physics implied by (13) by looking at the slow motion regime

$$L = \frac{1}{2} m \dot{\varphi}^2 - V_s(\varphi) + \mathcal{O}(\dot{\varphi}^4) , \quad (15)$$

with  $\varphi$  another radial variable related to  $r$  by<sup>1</sup>  $d\varphi = dr r^{3/2} f(r)^{-3/4}$ , matching to the zero-mode of the Higgs field in the dual SYM theory by the relation  $m\varphi^2 = V\phi^2$ . The resulting effective static potential takes the form

$$V_s(\varphi) = m \left[ r^3 \sqrt{f(r)} + \varepsilon (r_0^4 - r^4) \right] , \quad (16)$$

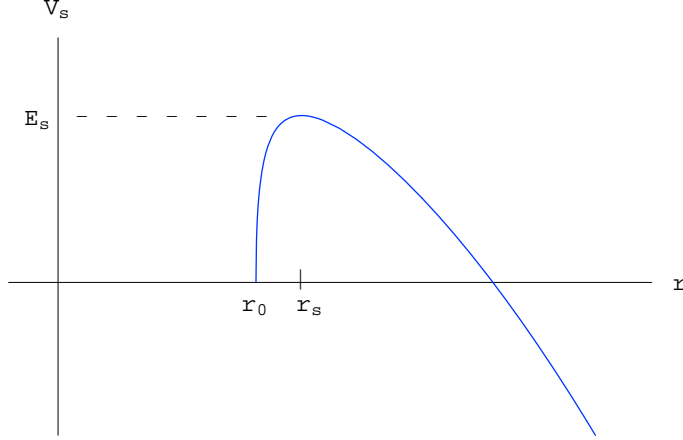
where the functional dependence  $r(\varphi)$  is implicitly understood. Asymptotically, as  $r \rightarrow \infty$  we have  $\varphi \propto r$  and the static potential for branes ( $\varepsilon = 1$ ) approaches the tachyonic regime  $V_s(r) \rightarrow -m r^2/2$ . We thus recover the leading tachyonic potential at large values of the Higgs field, the powers of  $r^4 \propto \varphi^4$  canceling out because of the BPS property of the D3-branes. Conversely, for antibranes ( $\varepsilon = -1$ ) the potential shows a quartic rise at large radius.

For generic values of  $r_0$ , the function  $V_s(r)$  is tangent to a vertical line at the horizon, indicating that the naive low-velocity approximation breaks down in the near-horizon region. For this reason, we shall not study the detailed near-horizon dynamics in terms of (15). Despite this fact, it will be shown that some properties of the motion are correctly captured by the static potential, such as the fact that neither branes nor antibranes can propagate with negative energy in the vicinity of the horizon. Branes of arbitrarily negative energy can exist, but their motion has a turning point ‘on the other side of the barrier’ (cf. figure 1).

Another exact property of the static potential turns out to be the location of the maximum,  $r_s$ , and the corresponding ‘sphaleron energy’  $E_s \equiv V_s(r_s)$ , characterizing the

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<sup>1</sup>See [7] for the general form of such field redefinitions.



**Figure 1:** Picture of the static potential  $V_s(r)$  for probe D-branes, showing the local maximum at the ‘sphaleron’ point  $r_s$ . Near-horizon D-branes with energy within the interval  $0 = V_s(r_0) < E < V_s(r_s) = E_s$  must tunnel through the barrier in order to escape, whereas D-branes with energy above  $E_s$  run away classically.

minimum energy for which the asymptotic motion has no turning points above the horizon. The sphaleron energy is related to the location of the maximum by the equation

$$E_s = \frac{1}{2} m \left( r_0^4 + r_0^2 - \frac{3}{2} r_s^2 \right) .$$

In the high-temperature limit,  $r_0 \gg 1$ , the barrier widens as  $r_s \rightarrow (r_0^8/2)^{1/6}$  and its height grows as  $E_s \rightarrow \frac{1}{2} m r_0^4$ . In the low-temperature limit, the barrier turns off as  $r_s \rightarrow r_0 \rightarrow 1/\sqrt{2}$  and  $E_s \rightarrow 0$ . In particular, the static potential at zero temperature,  $V_s(r)|_{T=0} = \frac{1}{2} m (\frac{1}{2} - r^2)$ , is monotonically decreasing for all radii above the horizon, showing that the barrier has all but disappeared at  $T = 0$ . This fact suggests that perhaps the barrier is not efficient in containing the branes for sufficiently low temperatures.

The ‘permeability’ of the barrier depends in practice on the typical energy of the branes impinging on it. At weak coupling, this is dictated by the thermal distribution of the scalar field configurations in the SYM theory. At strong coupling, the static potential depicted in figure 1 does not extend by itself to the origin of field space. Instead, we must view the horizon as providing the thermal-state initial condition for the motion of the branes in the region  $r \geq r_0$ . Hence, we will adopt the physical prescription that D-branes are emitted by the horizon with a Hawking spectrum. This means that the rate of ‘fragmentation’ in the pattern  $SU(N) \rightarrow SU(N-1) \times U(1)$  must be computed as the convolution of the Hawking rate with the ‘grey-body’ factor resulting from the quantum barrier penetration.

While these comments serve to frame the spirit of our computation, a number of remarks are in order regarding the detailed implementation. First, the nonrelativistic approximation implicit in  $V_s(r)$  breaks down in the vicinity of the horizon, so that the

WKB approximation to the decay rate must be implemented at the relativistic level. Second, the height of the static barrier is of order  $E_s = \mathcal{O}(mr_0^4)$ , which coincides in order of magnitude with the size of the chemical potential  $|\mu_N|$  (except at very low temperatures). This implies that the D-branes are emitted with a typical energy of order  $E_s$ , calling into question the efficiency of the barrier in containing the decay. In other words, the metastable character of the black hole is potentially sensitive to the leading finite- $N$  effects on the energetic balance of the emission process. Finally, the height of the barrier is proportional to the brane mass parameter,  $m$ , so that relativistic effects associated to pair-production compete with the standard tunneling processes and deserve specific study. In what follows we address each of these questions separately, keeping in mind the basic physical picture suggested by figure 1.

### 3.1 Non-linear dynamics

We now describe the qualitative properties of the brane motion in the *exterior* of the topological black holes, i.e. for radial coordinates  $r \geq r_0$ , including all nonlinear effects implied by the Lagrangian (13). The conserved canonical energy

$$E = \dot{r} \frac{\partial L}{\partial \dot{r}} - L, \quad (17)$$

satisfies

$$\frac{E}{m} + \varepsilon (r^4 - r_0^4) = \frac{r^3 f(r)}{\sqrt{f(r) - \dot{r}^2 / f(r)}}. \quad (18)$$

Squaring this equation and solving for  $\dot{r}$  we find a conveniently intuitive description of the dynamics as a zero-energy motion in the effective non-relativistic problem

$$\dot{r}^2 + V_{\text{eff}}(r) = 0, \quad (19)$$

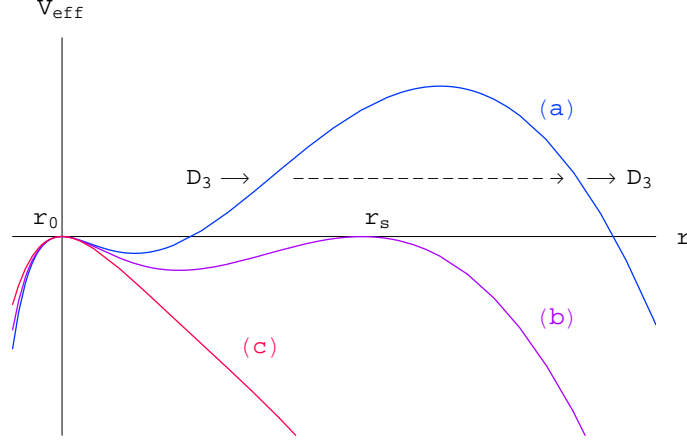
where the effective potential is given by

$$V_{\text{eff}}(r) = f(r)^2 \left[ \frac{r^6 f(r)}{\left( \frac{E}{m} + \varepsilon (r^4 - r_0^4) \right)^2} - 1 \right]. \quad (20)$$

Since this potential problem is obtained by squaring (17), some information about signs is lost in the process, and must be recovered from (18). Positiveness of the right-hand side of (18) implies that necessarily  $r^4 > r_0^4 - E/m$  for branes and  $r^4 < r_0^4 + E/m$  for antibranes. Therefore, the potential  $V_{\text{eff}}(r)$  with  $\varepsilon = 1$  describes at the same time branes of energy  $E$  moving ‘to the right of the pole’ and antibranes of energy  $-E$  moving ‘to the left of the pole’.

With no loss of generality, we then concentrate on the case  $\varepsilon = 1$ . Expanding the potential at large radius we find the universal asymptotic behavior  $V_{\text{eff}}(r \rightarrow \infty) \rightarrow -r^2$  as expected. On the other hand, for large and negative values of  $\omega$  we are always guaranteed a positive pole  $r_p = (r_0^4 - E/m)^{1/4}$  at large radius, showing that  $V_{\text{eff}}$  must have a zero at large radius of order  $r_+ > r_p$ , the turning point for the runaway trajectories.



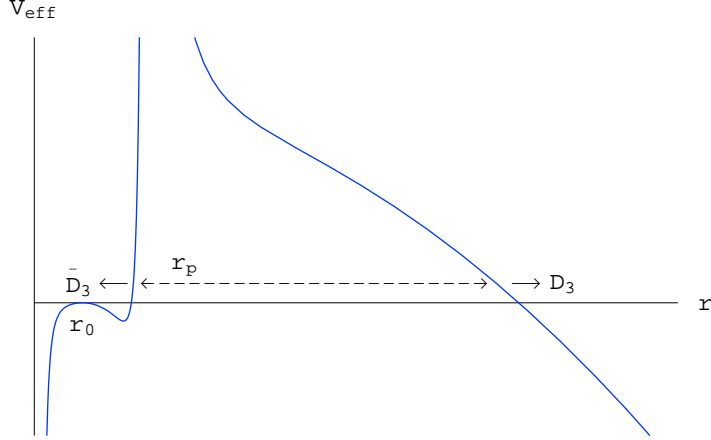


**Figure 2:** A picture of the non-linear effective potential just above the threshold of D-brane radiation. Curve (a) corresponds to  $0 < E < E_s$ , with curve (b) giving the *sphaleron* case,  $E = E_s$ . Finally, for case (c), with  $E > E_s$ , there is no barrier, and the potential becomes monotonically decreasing for  $E \gg E_s$  at all radii outside the horizon. In case (a) D3-branes emitted thermally from the horizon at  $r = r_0$  can tunnel across the barrier and subsequently run away to the asymptotic region. As  $T \rightarrow 0$ , we have  $E_s \rightarrow 0$  and the barrier tends to disappear.

As advanced in the previous section, we have two qualitatively different situations depending on whether the canonical energy of the probe brane is positive or negative. For  $E < 0$  the pole is in the physical region and only antiD-branes can propagate in the immediate vicinity of the horizon. Conversely, for  $E > 0$  the pole is formally ‘inside’ the black hole and thus no antiD-branes can propagate outside the horizon, whereas D-branes can actually fall into the black hole, if found in its vicinity.

Away from extremality, that is to say for  $\mu > -1/4$ , the profile function  $f(r)$  is positive for all  $r > r_0$  and has a simple zero at  $r = r_0$ . Therefore, both the potential and the first derivative vanish at the horizon, with a negative value of the second derivative, for all  $E \neq 0$ . It follows that the effective potential starts with a gentle quartic descent in the vicinity of the horizon, which is a region of allowed classical motion for either antibranes ( $E < 0$ ) or branes ( $E > 0$ ). The marginal situation  $E = 0$  has the pole cancelled out by the zero of  $f(r)$  so that neither branes nor antibranes can propagate near  $r_0$  in this case, except for the extremal black hole,  $\mu = -1/4$ , on which branes can still fall.

For  $E \gg 0$  the potential is monotonically decreasing for  $r \geq r_0$ . However, near the pole threshold, within an interval  $0 < E < E_s$  a finite barrier develops in the non-extremal case ( $\mu > -1/4$ ). At the ‘sphaleron’ energy,  $E_s$ , there is a local maximum at  $r_s > r_0$  with exactly vanishing potential  $V'_{\text{eff}}(r_s) = V_{\text{eff}}(r_s) = 0$ , so that  $r(t) = r_s$  is an unstable static trajectory. As  $E$  goes slightly below  $E_s$  the local maximum becomes positive and the barrier develops until the pole gets superimposed on the barrier for  $E < 0$ . One can explicitly check that this definition of the sphaleron point coincides with the one given above in terms of the static potential. Finally, as  $E \ll 0$  the pole at  $r_p \sim (r_0^4 - E/m)^{1/4}$  migrates far away from the horizon, accompanied by zeros of the



**Figure 3:** The effective potential  $V_{\text{eff}}$  below threshold, for  $E < 0$ , has a pole at  $r_p > r_0$ . D-branes can only propagate as zero-energy motions to the right of the second turning point, whereas antibranes with energy  $\bar{E} = -E > 0$  can be trapped between the horizon and the first turning point, to the left of the pole. The analog of the Schwinger pair production feeds such brane-antibrane pairs at the expense of the background RR field.

effective potential  $V_{\text{eff}}(r_{\pm}) = 0$ , with  $r_- = \mathcal{O}(r_p)$  and  $r_+ = \mathcal{O}(r_p^2)$ .

### 3.2 Energy conventions and Hawking spectrum

As mentioned in the previous section, the marginal height of the barrier, compared to the typical energy of D3-branes, requires a careful treatment of the different sources of energy differences in the emission process. We shall assume the physical boundary condition at the horizon that D-branes are emitted by the black hole at the appropriate Hawking rate. Lacking a complete theory of Hawking radiation for D-brane objects,<sup>2</sup> we will adopt here a physical prescription, demanding that the decay rate be proportional to the ratio of density of states before and after the emission of a D-brane,

$$\Gamma_H \propto \exp(\Delta S) , \quad (21)$$

where  $\Delta S = S_f - S_i$  for the initial and final black hole states. If we emit a single D3-brane of energy  $\omega$ , we have  $\Delta S = S(N-1, M-\omega) - S(N, M)$ . Using (7) we find, to leading order in the  $1/N$  expansion, the expected exponential rate

$$\Gamma_H(\omega) \propto \exp(-\beta\omega + \beta\mu_N) , \quad (22)$$

where  $\mu_N$  is given by (9). We can also consider the emission of antiD-branes, in which case the chemical potential has the opposite sign,  $\mu_{\bar{N}} = -\mu_N$ . Notice that  $\mu_N$  is negative, suppressing the emission of D-branes and enhancing the emission of antiD-branes.

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<sup>2</sup>It might be possible to develop a formalism based on the ideas of [13].

We shall apply the relation (22) only when it is semiclassical, i.e.  $\Gamma_H \ll 1$ . When the energy and temperature parameters are taken to extremes, so that the exponential approximation (22) gives  $\exp(-\beta\omega + \beta\mu_N) \sim 1$ , we assume the rate to be controlled, in order of magnitude, by the temperature:  $\Gamma_H \sim T$ , up to a dimensionless  $\mathcal{O}(1)$  function of  $T\ell$ . It would be interesting to determine if this function is smooth in the vicinity of the origin, which would ensure a power-law suppression of the decay rate in the extremal  $T \rightarrow 0$  limit. Such a study is however beyond the crude semiclassical methods used here, since one can expect subtleties related to the superradiant character of the modes with  $\omega < \mu_N$ .

In order to determine the complete decay rate, we need to relate the energy  $\omega$ , defined in terms of the thermodynamics of the gauge theory, at fixed  $V$  and  $\ell$ , to the canonical energy  $E$  defined by equation (17). Let us rewrite the total energy of the black hole as

$$M = \frac{3V}{16\pi G} \left( \mu + \frac{1}{4} \right) = \frac{3N^2 V}{8\pi^2} \left( \mu + \frac{1}{4} \right), \quad (23)$$

where we have used the convention  $\ell = 1$  in the second equation. On emitting a D3-brane of energy  $\omega$ , the black hole loses one unit of RR charge, so that  $\Delta N = N_f - N_i = -1$  and

$$-\omega = M_f - M_i \approx \Delta N \frac{\partial M}{\partial N} \Big|_{\mu} + \Delta \mu \frac{\partial M}{\partial \mu} \Big|_N = -\frac{6VN}{8\pi^2} \left( \mu + \frac{1}{4} \right) + \frac{3V}{16\pi G} \Delta \mu, \quad (24)$$

here  $\Delta \mu = \mu_f - \mu_i$  is the difference of ADM mass parameters as the emission takes place, which determines the ADM mass of the brane:

$$-\omega_{\text{ADM}} = \frac{3V}{16\pi G} \Delta \mu. \quad (25)$$

Hence, we find the relation

$$\omega = \omega_{\text{ADM}} + \frac{2}{N} M = \omega_{\text{ADM}} + \frac{3}{2} m \left( r_0^4 - r_0^2 + \frac{1}{4} \right). \quad (26)$$

It remains now to relate the ADM mass to the canonical mass, a task that we shall undertake by adapting the results of reference [16]. In order to proceed, we recall that the ADM formalism treats the branes as codimension-two defects in the effective five-dimensional gravitational theory on  $\text{AdS}_5$ , with fixed value of the Newton's constant  $G$ . On the other hand, after reduction on the  $\mathbf{S}^5$ , the RR form  $C_4$  gives rise to a five-dimensional cosmological constant with a quantized value, jumping by an amount of  $\mathcal{O}(1/N)$  on crossing a D3-brane. One can then solve Einstein's equations in the defect approximation, using Israel's junction conditions [14] (see also [15, 16]) for a metric defined in two AdS patches: an AdS black hole metric of type (1) on the 'exterior' of the brane, characterized by a profile function  $f_i(r) = r^2/\ell_i^2 - 1 - \mu_i/r^2$ , and an analogous interior metric, with profile function  $f_f(r) = r^2/\ell_f^2 - 1 - \mu_f/r^2$ . The two metrics are matched at the world-volume of the D3-brane, with induced metric

$$ds_{\text{induced}}^2 = -d\tau^2 + r(\tau)^2 d\Sigma_3^2, \quad (27)$$

with  $r(\tau)$  the trajectory of the brane in terms of its proper time. For any such defect of tension  $\sigma$  the junction conditions imply

$$\left[ f_f(r) + \left( \frac{dr}{d\tau} \right)^2 \right]^{1/2} - \left[ f_i(r) + \left( \frac{dr}{d\tau} \right)^2 \right]^{1/2} = \frac{8\pi G}{3} \sigma r(\tau) . \quad (28)$$

The relation between the asymptotic time appearing in (1) and the proper time of (27) can be found by matching the exterior metric and the induced metric, with the result

$$\frac{dt}{d\tau} = \sqrt{\frac{1}{f(r)} + \frac{1}{f(r)^2} \left( \frac{dr}{d\tau} \right)^2} ,$$

where we denote  $f_i(r) \equiv f(r)$ . Upon using this relation and squaring (28) twice we can represent the motion in the form (19) with the potential (cf. [16])

$$\tilde{V}_{\text{eff}}(r) = f(r)^2 \left[ \frac{r^6 \sigma^2 V^2 f(r)}{(\omega_{\text{ADM}} + q V r^4)^2} - 1 \right] , \quad (29)$$

where the effective charge  $q$  is given by

$$q = \frac{3}{16\pi G} \left( \frac{1}{\ell_f^2} - \frac{1}{\ell_i^2} \right) - \frac{4}{3} \pi G \sigma^2 .$$

The potential  $\tilde{V}_{\text{eff}}(r)$  matches exactly the  $\varepsilon = 1$  effective potential  $V_{\text{eff}}(r)$  under the natural BPS identification  $q = \sigma = \mathcal{T}_{\text{D3}}$ , and the further additive map of energy parameters:

$$\omega_{\text{ADM}} = E - m r_0^4 . \quad (30)$$

Hence, combining (26) and (30) we finally get

$$\omega = E + \frac{1}{2} m \left( r_0^4 - 3r_0^2 + \frac{3}{4} \right) . \quad (31)$$

As a result, we may rewrite the semiclassical Hawking rate of D3-brane emission in terms of the canonical energy  $E$  as

$$\Gamma_H(E) \propto e^{-\beta(E-E_0)} , \quad (32)$$

where the threshold energy  $E_0$  is given by

$$E_0 = \mu_N - \frac{1}{2} m \left( r_0^4 - 3r_0^2 + \frac{3}{4} \right) = -m(r_0^4 - r_0^2) , \quad (33)$$

where we have used the formula  $\mu_N = -\frac{1}{2} m \left( r_0^2 - \frac{1}{2} \right) \left( r_0^2 + \frac{3}{2} \right)$ . The critical energy  $E_0$  is large and negative for large temperature, and vanishes at  $r_0 = 1$ , corresponding to the special black hole isometric to pure AdS, with temperature  $T = (2\pi)^{-1}$ . On the other hand,  $E_0$  becomes positive for lower temperatures, approaching  $m/4$  for the extremal black hole. The semiclassical form (32) is therefore valid for  $E > E_0$  and, as indicated previously, we assume that  $\Gamma_H(E)$  loses the exponential suppression form for the window  $0 < E < E_0$ , which only opens up at low temperatures  $T < (2\pi)^{-1}$ .

## 4 Decay rates

In this section we estimate the leading tunneling effects across the barriers that D-branes may find outside the horizon. Since we have a very explicit description of the rigid motion in terms of the action (13), we use the WKB approximation at fixed canonical energy  $E$ . The wave function is given, in the leading approximation, by the *ansatz*

$$\Psi_E \propto \exp \left( -iEt + i \int^r p_{r'} dr' \right) \quad (34)$$

where the radial canonical momentum is defined as  $p_r = \partial L / \partial \dot{r}$ . The probability of barrier penetration is given, with exponential accuracy, by

$$\Gamma_E \sim \exp(-2 \operatorname{Im} W(E)) , \quad (35)$$

where  $W = I + Et$  is the so-called truncated action. The imaginary part in the classically forbidden region can be captured by the analytic continuation to the Euclidean signature  $t = -it_E$  and we find  $\operatorname{Im} W(E) \equiv W_E = I_E - Et_E$ , with  $I_E$  the Euclidean action

$$I_E = m \int dt_E \left[ r^3 \sqrt{f(r) + \frac{1}{f(r)} \left( \frac{dr}{dt_E} \right)^2} - \varepsilon (r^4 - r_0^4) \right] . \quad (36)$$

Finally, Euclidean trajectories correspond to motion in the effective problem

$$\left( \frac{dr}{dt_E} \right)^2 = 2V_{\text{eff}}(r) , \quad (37)$$

which is related to (19) by a formal sign flip of the effective potential. Using this equation in the formula for the Euclidean action, we find the convenient expression

$$W_E = m \int_{r_-}^{r_+} \frac{dr}{f(r)} \sqrt{r^6 f(r) - \left( r^4 - r_0^4 + \frac{\varepsilon E}{m} \right)^2} . \quad (38)$$

The integration domain is defined by the positivity of the square root argument, i.e.  $W_E$  vanishes when the turning points of the motion (37) degenerate at a single radius  $r_+ = r_- = r_s$ , defined by the largest solution  $r_s > r_0$  of  $V_{\text{eff}}(r_s) = V'_{\text{eff}}(r_s) = 0$  (the horizon itself is always a solution of these equations). This happens at the sphaleron energy,  $E_s = V_s(r_s)$ , which equals the value of the static potential evaluated at the static trajectory  $r(t_E) = r_s$ .

For  $\varepsilon = 1$  and energies in the range  $0 \leq E \leq E_s$  the tunneling exponent  $W_E$  governs the rate of barrier penetration for D-branes of energy  $E$ . For  $E > E_s$  the barrier disappears and  $W_E(E > E_s) = 0$ . On the other hand, for energies below the critical value  $E < 0$  the barrier features a pole at  $r_p = (r_0^4 - E/m)^{1/4}$ , in the physical region between the two turning points  $r_0 < r_- < r_p < r_+$ . Notice however that the expression (38) has no singular behavior across the pole and has, in fact, quite a smooth dependence on the energy  $E$ .

## 4.1 Pair creation

For  $E < 0$ , the Euclidean trajectories solving (37) correspond to D-branes of energy  $E$  propagating in the interval  $r_p < r < r_+$  and to antiD-branes of energy  $-E$  propagating in the interval  $r_- < r < r_p$ . Hence, we may interpret the solutions as the Euclidean description of the brane-antibrane nucleation process, similar to Schwinger's description of an electric field decay by  $e^+e^-$  emission [18]. The D-brane member of the pair emerges at  $r_+$  and falls to infinity, whereas the antiD-brane emerges at  $r_-$  and falls towards the black hole.

The analogy with the Schwinger process can be made quite literal. The electron Lagrangian in the presence of a constant electric field in one dimension reads

$$L_e = -m_e \sqrt{1 - \dot{x}^2} + e \mathcal{E} x . \quad (39)$$

Performing the same manipulations for this system we find an effective potential description  $\dot{x}^2 + V_e(x) = 0$ , with

$$V_e(x) = \frac{m_e^2}{(E + e \mathcal{E} x)^2} - 1 , \quad (40)$$

which, with the exception of the warping effects, has essentially the same structure as our brane potentials, including the occurrence of poles and the rule that electrons propagate in the region  $x > -E/e\mathcal{E}$  and positrons do so in the region  $x < -E/e\mathcal{E}$ . The two turning points at  $x_{\pm} = -E/e\mathcal{E} \pm m_e/e\mathcal{E}$  are interpreted semiclassically as the nucleation positions of  $e^+e^-$  pairs in a Schwinger decay process of the electric field. For the  $e^+e^-$  potential (40), equation (38) gives the expected exponent  $2W_E = 2 \int_{x_-}^{x_+} dx \sqrt{m_e^2 - (e\mathcal{E}x)^2} = \pi m_e^2/e\mathcal{E}$  controlling the Schwinger effect amplitude.

We can estimate the WKB factor (38) for very large temperatures,  $r_0 \gg 1$  by noticing that the turning points, defined by the vanishing of the square root in the integrand (38) are given by

$$r_- \approx \frac{r_p^2}{(2r_p^4 - r_0^4)^{1/4}} , \quad r_+ \approx (2r_p^4 - r_0^4)^{1/2} , \quad (41)$$

in this limit. In these expressions, we denote  $r_p = (r_0^4 - E/m)^{1/4}$  the pole position, and we recall that  $E < 0$  in our conventions. At high temperatures,  $r_0 \gg 1$ , we have  $r_+ \gg r_-$ , and the integral in (38) is dominated by the high endpoint, so that we have

$$W_E \approx m(2r_p^4 - r_0^4) \int_0^1 dx \sqrt{1 - x^2} \approx \frac{\pi}{4} m (r_0^4 - 2E/m) . \quad (42)$$

Using the asymptotic relation  $r_0 \sim \pi T \gg 1$ , we find an amplitude  $\exp(-2W_E)$  with

$$2W_E \approx \frac{\pi^3}{4} N V T^4 + \pi |E| , \quad (43)$$

where the second term is only to be kept when it dominates over the first, i.e. we have an amplitude of order  $\exp(-\pi|E|)$  for energies very large and negative,  $-E \gg 1$ , so that

$r_p \gg r_0$ . The complete Schwinger amplitude must be integrated over all negative energies or, equivalently over all pole locations  $r_p \geq r_0$ . In view of (43), this integral is dominated by the endpoint at zero energy,  $E = 0$ , which yields a total Schwinger amplitude at high temperatures

$$\Gamma_S \sim \exp \left( -\frac{\pi^3}{4} N V T^4 \right) , \quad (44)$$

as always, in units  $\ell = 1$ .

## 4.2 Thermal emission of D-branes

For  $E > 0$  the Schwinger process does not take place, as the pole migrates behind the horizon. Instead, we have a more standard process of tunneling across the potential barrier, with the initial state being fed by the Hawking radiation law. The barrier is only effective for energies in the window  $0 < E < E_s$ . Our analysis of the previous section showed that the correct rate of D3-branes impinging on the barrier is given by

$$\Gamma_H(E) \sim e^{-\beta(E-E_0)} , \quad (45)$$

provided  $E > E_0 = -m(r_0^4 - r_0^2)$ . In the window  $0 < E < E_0$ , which only opens up for  $T < (2\pi)^{-1}$ , we assume that the decay rate is controlled by the temperature  $\Gamma_H(E) \sim T$  and it is not expected to depend exponentially on the energy of the brane or the charge  $N$  of the black hole.

We see that for all temperatures for which  $E_s > E_0$  the metastability of the black hole is protected either by the exponential suppression of the Hawking rate or by the presence of the external barrier. This is the case at large temperatures. On the other hand, at low temperatures  $E_s$  vanishes, whereas  $E_0$  approaches the positive value  $m/4$ , resulting in the opening of a window,  $E_s < E < E_0$ , for which the D-branes escape without exponential suppression from neither the Hawking rate nor the grey body factor. The critical temperature at which metastability is lost is of order  $T_c \approx 0.12$  in units  $\ell = 1$ , and can be found by solving numerically the equation  $E_0 = E_s$ .

The breakdown of metastability for  $T < T_c$  can be suggestively interpreted as the strong-coupling counterpart of the low- $T$  thermal mass being dominated by the tachyonic mass at weak coupling.

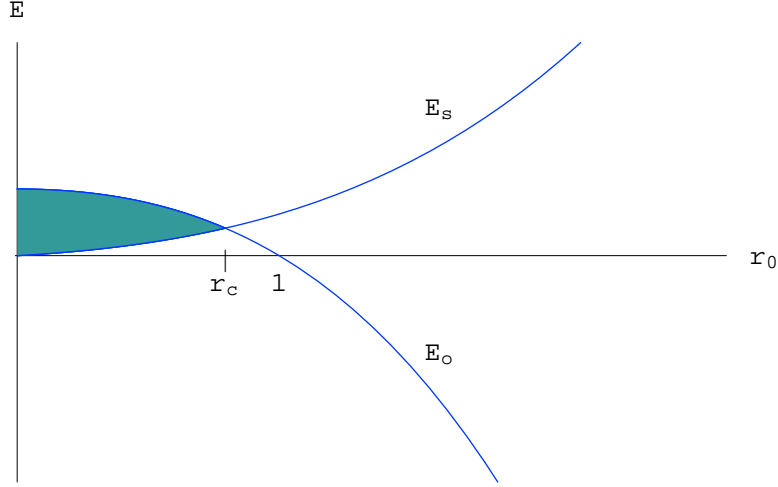
At  $T > T_c$  the black hole is indeed metastable, and the total emission rate can be approximated by the convolution

$$\Gamma_{\text{thermal}} \sim \int_0^\infty dE \Gamma_H(E) e^{-2W_E} . \quad (46)$$

Using the exponential form and performing a standard saddle-point evaluation (see for example [17]) we find

$$e^{\beta E_0} \int dE e^{-\beta E - 2W_E} \sim e^{\beta E_0} e^{-I_E(\beta)} ,$$

where  $I_E(\beta)$  is the value of the Euclidean action at a periodic trajectory solving (37) with period  $\beta$ . The period of such thermal bounces is larger the larger is the barrier. The



**Figure 4:** Comparison of the threshold energy  $E_0$  and the sphaleron energy  $E_s$  as a function of temperature, or rather horizon radius in this picture. The shaded region, at horizon radii  $r \leq r_c \approx 0.93$ , represents the window of D-brane energies  $E_s < E < E_0$  for which metastability is lost.

minimum period is attained when the barrier disappears, and is given by  $2\pi/\Omega_s$ , where  $\Omega_s^2 = V''_{\text{eff}}(r_s)$  is the second derivative of the effective potential at the sphaleron value of the energy  $E_s$ . Hence, for  $\beta < 2\pi/\Omega_s$  there are no possible bounces with that period and the rate is dominated by the *static* solution, corresponding to the constant trajectory  $r(t_E) = r_s$  for the potential at energy  $E_s$ . Computing the action for this solution one finds

$$e^{-I_E(\text{sphaleron})} = e^{-\beta V_s(r_s)} = e^{-\beta E_s} . \quad (47)$$

At large temperatures the sphaleron frequency  $\Omega_s = \mathcal{O}(1)$ , so that for any large temperature  $T \gg 1$  in units of the AdS radius, the rate will be given by the sphaleron approximation

$$\Gamma_{\text{thermal}}|_{T \gg 1} \sim e^{-\beta(E_s - E_0)} \sim \exp\left(-\frac{\pi^2}{4} N V T^3\right) . \quad (48)$$

Incidentally, this has the same order of magnitude, within exponential accuracy, as the high-energy endpoint contribution, corresponding to the high-energy Hawking tail without grey-body suppression,  $\int_{E_s}^{\infty} dE e^{-\beta(E - E_0)} \sim e^{-\beta(E_s - E_0)}$ .

## 5 Conclusions

We have analyzed the metastability of the  $SU(N)$  symmetric state of  $\mathcal{N} = 4$  SYM theory on a compact three-dimensional hyperboloid, using the AdS bulk description of the system at large  $N$  and large values of the 't Hooft coupling,  $\lambda = g_{\text{YM}}^2 N \gg 1$ . The zero-mode energy of the Higgs fields is unbounded below in this theory, whereas one



expects the  $SU(N)$ -symmetric state at vanishing values of the Higgs fields to be locally stable at finite temperature. We test this expectation in the supergravity approximation and estimate the corresponding rates for the decay of the  $SU(N)$ -symmetric metastable state into  $SU(N-1)$ , times a runaway  $U(1)$  factor.

Our main results are as follows. At large temperatures  $T \gg (2\pi\ell)^{-1}$  the thermal states are metastable, with an exponentially suppressed decay rate of order

$$\Gamma_{\text{thermal}}|_{2\pi\ell T \gg 1} \sim \exp\left(-\frac{\pi^2}{4}NV T^3\right), \quad (49)$$

and dominated by thermal excitation over the ‘grey-body’ barrier, although a competing effect with a smaller coefficient also manifests itself by quantum nucleation of brane-antibrane pairs, in an analog of Schwinger’s process,

$$\Gamma_{\text{S}}|_{2\pi\ell T \gg 1} \sim \exp\left(-\frac{\pi^2}{4}NV T^3(\pi\ell T)\right). \quad (50)$$

We see that the pair production effect is exponentially suppressed with respect to the thermal excitation over the barrier (at large temperatures), by an extra relative factor of  $\pi T\ell \gg 1$  in the exponent.

A critical temperature exists,  $T_c \approx (8.3\ell)^{-1} < (2\pi\ell)^{-1}$ , below which the thermal emission process loses its exponential suppression with the RR charge  $N$ , becoming a much faster powerlike rate which renders the low-temperature black holes much less stable than their large-temperature counterparts. It would be interesting to improve the analysis of this low-temperature ‘superradiant’ regime to obtain an estimate of the powerlike behavior. The main difficulty in this task lies in the corrections to the Hawking rate of D-branes, rather than the next-to-leading corrections in the calculation of barrier penetration. It is interesting that the special topological black hole at  $r_0 = 1$ , locally isometric to pure AdS, falls into the metastable domain according to our results.

We conclude that a hot black hole decays extremely slowly until the temperature drops below the critical value, set by the AdS curvature, when the process accelerates to a rate of order  $T$ . Eventually the value of  $N$  and  $\lambda = g_{\text{YM}}^2 N$  gradually decrease in the interior geometry that is left over. When the ‘interior’ value of the ’t Hooft coupling reaches  $\mathcal{O}(1)$ , a matching to a weakly-coupled description is required, since the strong curvature prevents the use of geometrical methods (see [19] for preliminary results in this direction). It would be interesting to study whether the lack of metastability at low temperatures that we found here extends to the perturbative Yang–Mills domain, by an explicit computation of the thermal effective potential on the compact hyperboloid. Such an analysis would surely shed light on the use of these backgrounds in the program spelled out [10].

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